Relaxation of variational models in plasticity
and nonlinear elasticity

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Topics:

1. Introduction
2. Lower semicontinuity and quasiconvexity [Mor52], modern presentations in [Dac89, Mül99, Rou97].
3. Properties of quasiconvex functions, rank-one convexity, [Dac89, Mül99, Rou97].
4. Polyconvexity, existence [Bal77, Šve92].
5. Relaxation theory. [Dac81, Dac82], see also [Dac89, Mül99, Rou97].
6. Relaxation in linear plasticity. Relaxation [CO05], simulations [CHO07].
7. Nematic elastomers. Qc envelope [DD02], simulations [CDD02], model and physical background [WT03].
8. Single-slip plasticity. Model and physical background [OR99, CHM02], qc envelopes [CT05, Con06].
9. Relaxation with det = 1: Piecewise affine construction [Con08], convex integration [MŠ99], relaxation [CD14], soft case $p < n$ [Bal82, BM84, KRW13b, KRW13a]
10. Relaxation with det > 0: [CD14], for $p = \infty$ see [BK13].
11. Single-slip plasticity in the limit of rigid elasticity. Γ-Convergence [CDK11], extended div-curl Lemma [CDM11].

[note: many are available from http://www.iam.uni-bonn.de/aaa2/].

References

[Bal82] J. M. Ball. Discontinuous equilibrium solutions and cavitation in nonlinear elasticity. *Philos. Trans. Roy. Soc. London Ser. A*, 306:557–611, 1982.

[Bal77] J. M. Ball. Convexity conditions and existence theorems in nonlinear elasticity. *Arch. Rational Mech. Anal.*, 63:337–403, 1976/77.

[BK13] B. Benešová and M. Kružík. Characterization of gradient Young measures generated by homeomorphisms in the plane. *Preprint arXiv:1308.3377*, 2013.

[BM84] J. M. Ball and F. Murat. $W^{1,p}$-quasiconvexity and variational problems for multiple integrals. *J. Funct. Anal.*, 58:225–253, 1984.

[CD14] S. Conti and G. Dolzmann. On the theory of relaxation in nonlinear elasticity with constraints on the determinant. *Arch. Rat. Mech. Anal., online first*, 2014.

[CDD02] S. Conti, A. DeSimone, and G. Dolzmann. Soft elastic response of stretched sheets of nematic elastomers: a numerical study. *J. Mech. Phys. Solids*, 50:1431–1451, 2002.

[CDK11] S. Conti, G. Dolzmann, and C. Kreisbeck. Asymptotic behavior of crystal plasticity with one slip system in the limit of rigid elasticity. *SIAM J. Math. Anal.*, 43:2337–2353, 2011.

[CDM11] S. Conti, G. Dolzmann, and S. Müller. The div-curl lemma for sequences whose divergence and curl are compact in $W^{-1,1}$. *Comptes Rendus Math.*, 349:175–178, 2011.

[CHM02] C. Carstensen, K. Hackl, and A. Mielke. Non-convex potentials and microstructures in finite-strain plasticity. *R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci.*, 458:299–317, 2002.

[CHO07] S. Conti, P. Hauret, and M. Ortiz. Concurrent multiscale computing of deformation microstructure by relaxation and local enrichment with application to single-crystal plasticity. *Multiscale Modeling and Simulation*, 6:135–157, 2007.

[CO05] S. Conti and M. Ortiz. Dislocation microstructures and the effective behavior of single crystals. *Arch. Rat. Mech. Anal.*, 176:103–147, 2005.

[Con06] S. Conti. Relaxation of single-slip single-crystal plasticity with linear hardening. In P. Gumbsch, editor, *Multiscale Materials Modeling*, pages 30–35, Freiburg, 2006. Fraunhofer IRB.

[Con08] S. Conti. Quasiconvex functions incorporating volumetric constraints are rank-one convex. *J. Math. Pures Appliquees*, 90:15–30, 2008.

[CT05] S. Conti and F. Theil. Single-slip elastoplastic microstructures. *Arch. Rat. Mech. Anal.*, 178:125–148, 2005.

[Dac81] B. Dacorogna. A relaxation theorem and its application to the equilibrium of gases. *Arch. Rational Mech. Anal.*, 77:359–386, 1981.

[Dac82] B. Dacorogna. Quasiconvexity and relaxation of nonconvex problems in the calculus of variations. *J. Funct. Anal.*, 46:102–118, 1982.

[Dac89] B. Dacorogna. *Direct methods in the calculus of variations*, volume 78 of *Applied Mathematical Sciences*. Springer-Verlag, Berlin, 1989.

[DD02] A. DeSimone and G. Dolzmann. Macroscopic response of nematic elastomers via relaxation of a class of SO(3)-invariant energies. *Arch. Ration. Mech. Anal.*, 161:181–204, 2002.

[KRW13a] K. Koumatos, F. Rindler, and E. Wiedemann. Differential inclusions and Young measures involving prescribed jacobians. *Preprint arXiv:1312.1820*, 2013.

[KRW13b] K. Koumatos, F. Rindler, and E. Wiedemann. Orientation-preserving Young measures. *Preprint arXiv:1307.1007*, 2013.

[Mor52] C. B. Morrey, Jr. Quasi-convexity and the lower semicontinuity of multiple integrals. *Pacific J. Math.*, 2:25–53, 1952.

[MŠ99] S. Müller and V. Šverák. Convex integration with constraints and applications to phase transitions and partial differential equations. *J. Eur. Math. Soc. (JEMS)*, 1:393–442, 1999.

[Mül99] S. Müller. Variational models for microstructure and phase transitions. In F. Bethuel et al., editors, *Calculus of variations and geometric evolution problems*, Springer Lecture Notes in Math. 1713, pages 85–210. Springer-Verlag, 1999.

[OR99] M. Ortiz and E. A. Repetto. Nonconvex energy minimization and dislocation structures in ductile single crystals. *J. Mech. Phys. Solids*, 47:397–462, 1999.

[Rou97] T. Roubíček. *Relaxation in optimization theory and variational calculus*, volume 4 of *de Gruyter Series in Nonlinear Analysis and Applications*. Walter de Gruyter & Co., Berlin, 1997.

[Šve92] V. Šverák. Rank-one convexity does not imply quasiconvexity. *Proc. Roy. Soc. Edinburgh Sect. A*, 120:185–189, 1992.

[WT03] M. Warner and E. M. Terentjev. *Liquid Crystal Elastomers*. Oxford Univ. Press, 2003.